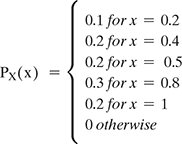
1. Given X be a discrete random variable with the following PMF



1. Find the range RX of the random variable X.

2. Find P(X ≤ 0.5)

3. Find P(0.25<X<0.75)

4. P(X = 0.2|X<0.6)

1. Ans:- The range of XX can be found from the PMF. The range of XX consists of possible values for XX. Here we have

RX={0.2,0.4,0.5,0.8,1}.RX={0.2,0.4,0.5,0.8,1}.

1. The event X≤0.5X≤0.5 can happen only if XX is 0.2,0.4,0.2,0.4, or 0.50.5. Thus,

|  |  |
| --- | --- |
| P(X≤0.5)P(X≤0.5) | =P(X∈{0.2,0.4,0.5})=P(X∈{0.2,0.4,0.5}) |
|  | =P(X=0.2)+P(X=0.4)+P(X=0.5)=P(X=0.2)+P(X=0.4)+P(X=0.5) |
|  | =PX(0.2)+PX(0.4)+PX(0.5)=PX(0.2)+PX(0.4)+PX(0.5) |
|  | =0.1+0.2+0.2=0.5=0.1+0.2+0.2=0.5 |

1. Similarly, we have

|  |  |
| --- | --- |
| P(0.25<X<0.75)P(0.25<X<0.75) | =P(X∈{0.4,0.5})=P(X∈{0.4,0.5}) |
|  | =P(X=0.4)+P(X=0.5)=P(X=0.4)+P(X=0.5) |
|  | =PX(0.4)+PX(0.5)=PX(0.4)+PX(0.5) |
|  | =0.2+0.2=0.4=0.2+0.2=0.4 |

1. This is a conditional probability problem, so we can use our famous formula P(A|B)=P(A∩B)P(B)P(A|B)=P(A∩B)P(B). We have

|  |  |
| --- | --- |
| P(X=0.2|X<0.6)P(X=0.2|X<0.6) | =P((X=0.2) and (X<0.6))P(X<0.6)=P((X=0.2) and (X<0.6))P(X<0.6) |
|  | =P(X=0.2)P(X<0.6)=P(X=0.2)P(X<0.6) |
|  | =PX(0.2)PX(0.2)+PX(0.4)+PX(0.5)=PX(0.2)PX(0.2)+PX(0.4)+PX(0.5) |
|  | =0.10.1+0.2+0.2=0.2=0.10.1+0.2+0.2=0.2 |

2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Find RX, RY, and the PMFs of X and Y.

2. Find P(X = 2,Y = 6).

3. Find P(X>3|Y = 2).

4. If Z = X + Y. Find the range and PMF of Z.

5. Find P(X = 4|Z = 8).

1. Ans:- We have RX=RY={1,2,3,4,5,6}RX=RY={1,2,3,4,5,6}. Assuming the dice are fair, all values are equally likely so

PX(k)={160for k=1,2,3,4,5,6otherwisePX(k)={16for k=1,2,3,4,5,60otherwise

Similarly for YY,

PY(k)={160for k=1,2,3,4,5,6otherwisePY(k)={16for k=1,2,3,4,5,60otherwise

1. Since XX and YY are independent random variables, we can write

|  |  |
| --- | --- |
| P(X=2,Y=6)P(X=2,Y=6) | =P(X=2)P(Y=6)=P(X=2)P(Y=6) |
|  | =16⋅16=136=16⋅16=136. |

1. Since XX and YY are independent, knowing the value of YY does not impact the probabilities for XX,

|  |  |
| --- | --- |
| P(X>3|Y=2)P(X>3|Y=2) | =P(X>3)=P(X>3) |
|  | =PX(4)+PX(5)+PX(6)=PX(4)+PX(5)+PX(6) |
|  | =16+16+16=12=16+16+16=12. |

1. First, we have RZ={2,3,4,...,12}RZ={2,3,4,...,12}. Thus, we need to find PZ(k)PZ(k) for k=2,3,...,12k=2,3,...,12. We have

|  |  |
| --- | --- |
| PZ(2)PZ(2) | =P(Z=2)=P(X=1,Y=1)=P(Z=2)=P(X=1,Y=1) |
|  | =P(X=1)P(Y=1) (since X and Y are independent)=P(X=1)P(Y=1) (since X and Y are independent) |
|  | =16⋅16=136=16⋅16=136; |
| PZ(3)PZ(3) | =P(Z=3)=P(X=1,Y=2)+P(X=2,Y=1)=P(Z=3)=P(X=1,Y=2)+P(X=2,Y=1) |
|  | =P(X=1)P(Y=2)+P(X=2)P(Y=1)=P(X=1)P(Y=2)+P(X=2)P(Y=1) |
|  | =16⋅16+16⋅16=118=16⋅16+16⋅16=118; |
| PZ(4)PZ(4) | =P(Z=4)=P(X=1,Y=3)+P(X=2,Y=2)+P(X=3,Y=1)=P(Z=4)=P(X=1,Y=3)+P(X=2,Y=2)+P(X=3,Y=1) |
|  | =3⋅136=112=3⋅136=112. |

1. We can continue similarly:

|  |  |
| --- | --- |
| PZ(5)PZ(5) | =436=19=436=19; |
| PZ(6)PZ(6) | =536=536; |
| PZ(7)PZ(7) | =636=16=636=16; |
| PZ(8)PZ(8) | =536=536; |
| PZ(9)PZ(9) | =436=19=436=19; |
| PZ(10)PZ(10) | =336=112=336=112; |
| PZ(11)PZ(11) | =236=118=236=118; |
| PZ(12)PZ(12) | =136=136. |

1. It is always a good idea to check our answers by verifying that ∑z∈RZPZ(z)=1∑z∈RZPZ(z)=1. Here, we have

|  |  |
| --- | --- |
| ∑z∈RZPZ(z)∑z∈RZPZ(z) | =136+236+336+436+536+636=136+236+336+436+536+636 |
|  | +536+436+336+236+136+536+436+336+236+136 |
|  | =1=1. |

1. Note that here we cannot argue that XX and ZZ are independent. Indeed, ZZ seems to completely depend on XX, Z=X+YZ=X+Y. To find the conditional probability P(X=4|Z=8)P(X=4|Z=8), we use the formula for conditional probability

|  |  |
| --- | --- |
| P(X=4|Z=8)P(X=4|Z=8) | =P(X=4,Z=8)P(Z=8)=P(X=4,Z=8)P(Z=8) |
|  | =P(X=4,Y=4)P(Z=8)=P(X=4,Y=4)P(Z=8) |
|  | =P(X=4)P(Y=4)P(Z=8) (since X and Y are independent)=P(X=4)P(Y=4)P(Z=8) (since X and Y are independent) |
|  | 16⋅1653616⋅16536 |
|  | =15=15. |

1. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

Ans:- P(X>15)=PX(16)+PX(17)+PX(18)+PX(19)+PX(20)=(106)(14)6(34)4+(107)(14)7(34)3+(108)(14)8(34)2+(109)(14)9(34)1+(1010)(14)10(34)0.

1. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

Ans:- We are looking at an interval of length 1.51.5 hours, so the number of customers in this interval is X∼Poisson(λ=1.5×10=15)X∼Poisson(λ=1.5×10=15). Thus,

|  |  |
| --- | --- |
| P(10<X≤15)P(10<X≤15) | =∑15k=11PX(k)=∑k=1115PX(k) |
|  | =∑15k=11e−1515kk!=∑k=1115e−1515kk! |
|  | =e−15[151111!+151212!+151313!+151414!+151515!]=e−15[151111!+151212!+151313!+151414!+151515!] |
|  | =0.4496 |

5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

Ans:-

|  |  |  |
| --- | --- | --- |
| PZ(k)PZ(k) | =P(X+Y=k)=P(X+Y=k) |  |
|  | =∑ki=0P(X+Y=k|X=i)P(X=i)=∑i=0kP(X+Y=k|X=i)P(X=i) | (law of total probability) (law of total probability) |
|  | =∑ki=0P(Y=k−i|X=i)P(X=i)=∑i=0kP(Y=k−i|X=i)P(X=i) |  |
|  | =∑ki=0P(Y=k−i)P(X=i)=∑i=0kP(Y=k−i)P(X=i) | (since X and Y are independent) (since X and Y are independent) |
|  | =∑ki=0e−ββk−i(k−i)!e−ααii!=∑i=0ke−ββk−i(k−i)!e−ααii! |  |
|  | =e−(α+β)∑ki=0αiβk−i(k−i)!i!=e−(α+β)∑i=0kαiβk−i(k−i)!i! |  |
|  | =e−(α+β)k!∑ki=0k!(k−i)!i!αiβk−i=e−(α+β)k!∑i=0kk!(k−i)!i!αiβk−i |  |
|  | =e−(α+β)k!∑ki=0(ki)αiβk−i=e−(α+β)k!∑i=0k(ki)αiβk−i |  |
|  | =e−(α+β)k!(α+β)k |  |

8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

* Ans:- If WW is the total weight, then W=X1+X2+⋯+XnW=X1+X2+⋯+Xn, where n=100n=100. We have

EWVar(W)=nμ=(100)(170)=17000,=100Var(Xi)=(100)(30)2=90000.EW=nμ=(100)(170)=17000,Var(W)=100Var(Xi)=(100)(30)2=90000.

Thus, σW=300σW=300. We have

P(W>18000)=P(W−17000300>18000−17000300)=P(W−17000300>103)=1−Φ(103)(by CLT)≈4.3×10−4.

9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.

Ans:- P(4≤Y≤6)=P(3.5≤Y≤6.5)(continuity correction)=P(3.5−526–√≤Y−526–√≤6.5−526–√)=P(−0.3062≤Y−526–√≤+0.3062)≈Φ(0.3062)−Φ(−0.3062)(by the CLT)=2Φ(0.3062)−1≈0.2405